

B.Sc. Semester - 5 (Remedial) (CBCS) Examination
March/April-2025 (NEW COURSE)
Mathematical Analysis -1 & Abstract Algebra -1(Core)

Time: 2:30 Hours

Marks: 70

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1 (A) Answer any TWO. (10)

- 1) Define a metric space. Let (X, d) be a metric space, show that closure of every subset of X is closed in X .
- 2) State and prove the Hausdorff principle for metric spaces.
- 3) Define Cantor set on $[0, 1]$. Show that a Cantor set is closed. Comment on the existence of minimum of Cantor set and if possible find it.

(B) Answer any ONE. (04)

- 1) (a) Prove or disprove: $\overline{A \cup B} = \overline{A} \cup \overline{B}$.
- (b) Is the set of rational numbers dense in the set of real numbers? Justify.
- 2) (a) Check whether $P = (3, \infty) \cup \{2\}$ is open or closed and justify.
- (b) Define boundary point. Find boundary points of set $P = (3, \infty) \cup \{2\}$.

Que-2 (A) Answer any TWO. (10)

- 1) Prove that any bounded function f defined on $[a, b]$ is R-integrable if and only if for every $\epsilon > 0$ there exist a partition P of $[a, b]$ such that $U(P, f) - L(P, f) < \epsilon$.
- 2) If functions f and g are R-integrable on $[a, b]$ then show that $f - g$ is also R-integrable on $[a, b]$.
- 3) For $0 \leq x \leq \frac{\pi}{2}$, show that $f(x) = \cos x$ is R-integrable and find $\int_0^{\frac{\pi}{2}} \cos x dx$.

(B) Answer any ONE. (04)

- 1) For $f(x) = \frac{1}{1+x}$, $x \in [0, 1]$ and partition $P = \left\{0, \frac{2}{7}, \frac{3}{7}, \frac{6}{7}, \frac{13}{14}, 1\right\}$, evaluate $U(P, f)$ and $L(P, f)$.
- 2) Solve $\int_0^1 (2x+1) dx$ using the definition of Riemann integration.

Que-3 (A) Answer any TWO. (10)

- 1) Show that $\int_0^{\frac{\pi}{4}} x \tan x \, dx \leq \frac{\pi}{8} \log_e 2$.
- 2) State and prove the first mean value theorem for Riemann integration.
- 3) Define a group. Show that $G = R - \{-1\}$ is a group under $*$ operation, defined by $a * b = a + b + ab \quad \forall a, b \in G$.

(B) Answer any ONE. (04)

- 1) Evaluate : $\lim_{n \rightarrow \infty} \frac{1}{n} \left[e^{\frac{3}{n}} + e^{\frac{6}{n}} + \dots + e^{\frac{3n}{n}} \right]$.
- 2) Prove that the set of all cube roots of unity form a commutative group with respect to the binary operation of multiplication of complex numbers.

Que-4 (A) Answer any TWO. (10)

- 1) State and prove Lagrange's theorem for a subgroup of a group.
- 2) Define a symmetric group and an alternating group. Prepare a group table for alternating group of symmetric group S_3 in usual notations.
- 3) Define : Normal Subgroup. Prove that A_3 is a Normal Subgroup of S_3 .

(B) Answer any ONE. (04)

- 1) Let G be a finite group of even order. Prove that there exist an element $a \neq e$ in G such that $a = a^{-1}$.
- 2) In a finite group G , prove that $o(bab^{-1}) = o(a) \quad \forall a, b \in G$.

Que-5 (A) Answer any TWO. (10)

- 1) Prove that an infinite cyclic group has exactly two generators.
- 2) Prove that the relation $\cong$ of isomorphism of groups is an equivalence relation.
- 3) State and prove Cayley's theorem.

(B) Answer any ONE. (04)

- 1) Show that $(R^+, \cdot) \cong (R, +)$.
- 2) Let G be a group and H be its subgroup. If index of H in G is 2 then prove that H is a normal subgroup of G .
